

Building up an Annuity

Lecture 4

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- 1 Definitions
- 2 The Effect of Time
- 3 Building up an Annuity
- 4 Examples
- 5 Assignment

Outline

- 1 Definitions
- 2 The Effect of Time
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Definitions

Definition (Annuity (Webster))

An **annuity** is a plan where a fixed amount of money that is paid in regular payments to a person over time.

- Typically, a retirement plan is an annuity – You invest over your working life and then withdraw from it monthly during retirement.
- One could establish an annuity to pay for a child's college education – You invest for 18 years and withdraw from it semiannually over the next 4 years.

Annuities

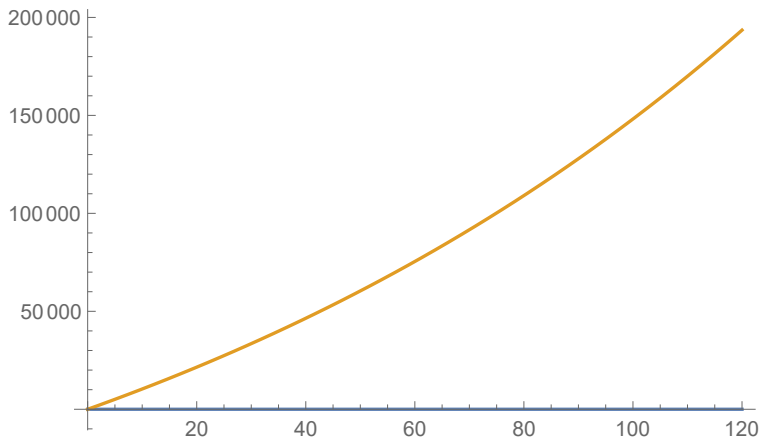
- An annuity has two stages.
 - The investment stage (building it up).
 - The withdrawal stage (drawing it down).
- During the investment stage, the balance grows.
- During the withdrawal stage, the balance diminishes.

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- Twenty years ago, the DJIA was at about 830.
- Now it is at about 26000.
- That represents an annual growth rate of 9%.

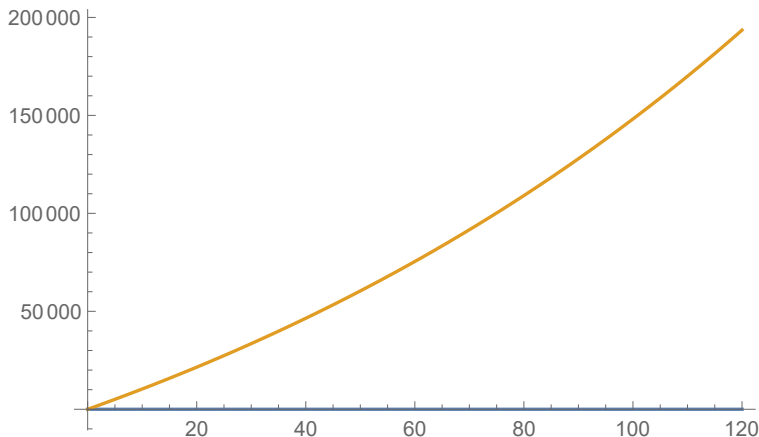
The Effect of Time



Invest \$1,000 per month at 9% for 10 years

Save up \$193,514.28.

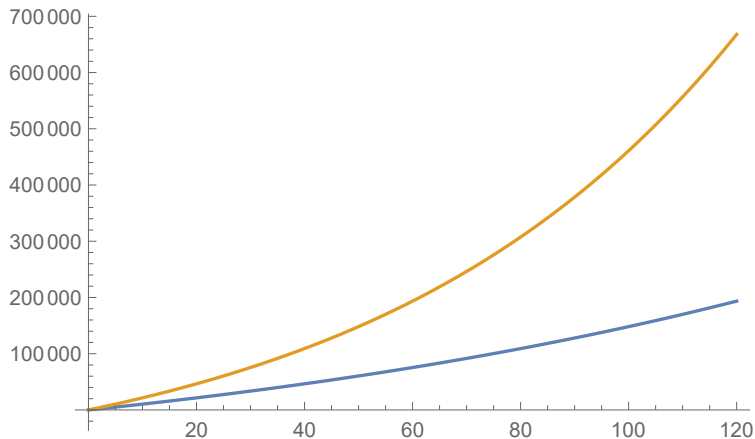
The Effect of Time



Invest \$1,000 per month at 9% for 10 years

Withdraw \$1,741.10 per month for 20 years

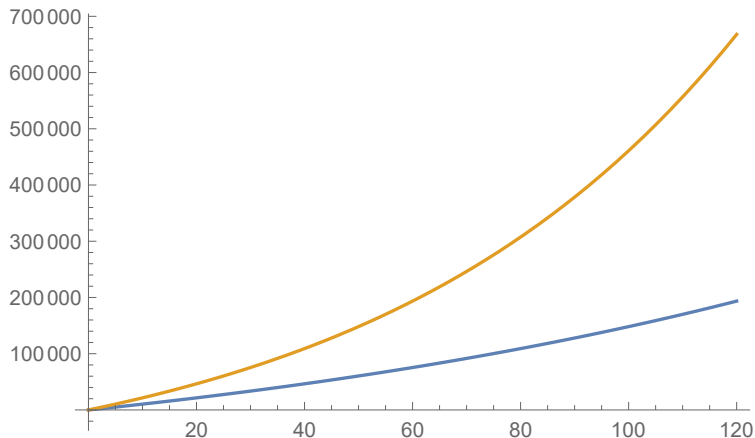
The Effect of Time



Invest \$1,000 per month at 9% for 20 years

Save up \$667,886.87.

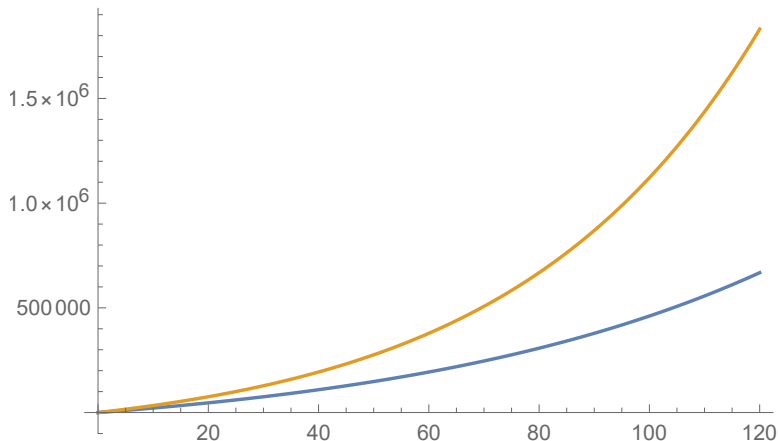
The Effect of Time



Invest \$1,000 per month at 9% for 20 years

Withdraw \$6,009.15 per month for 20 years

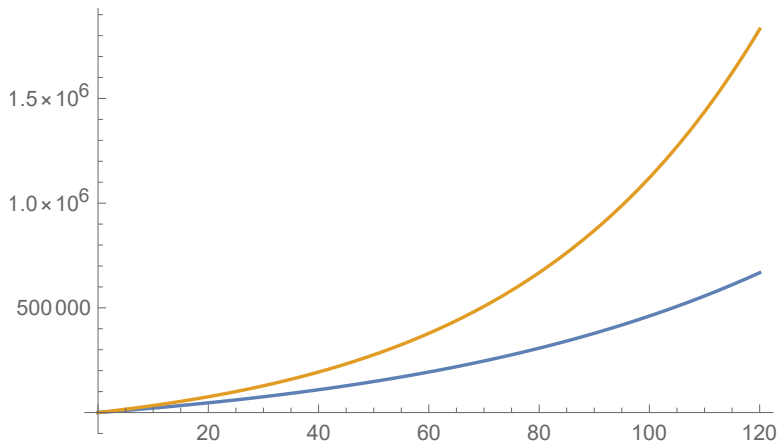
The Effect of Time



Invest \$1,000 per month at 9% for 30 years

Save up \$1,830,743.48.

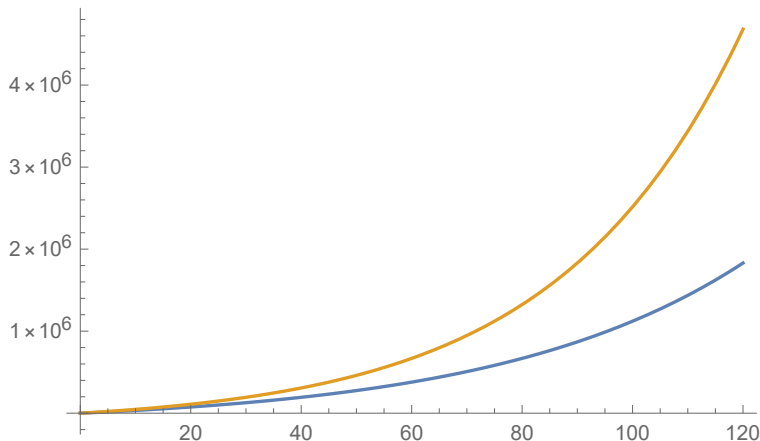
The Effect of Time



Invest \$1,000 per month at 9% for 30 years

Withdraw \$16,471.67 per month for 20 years

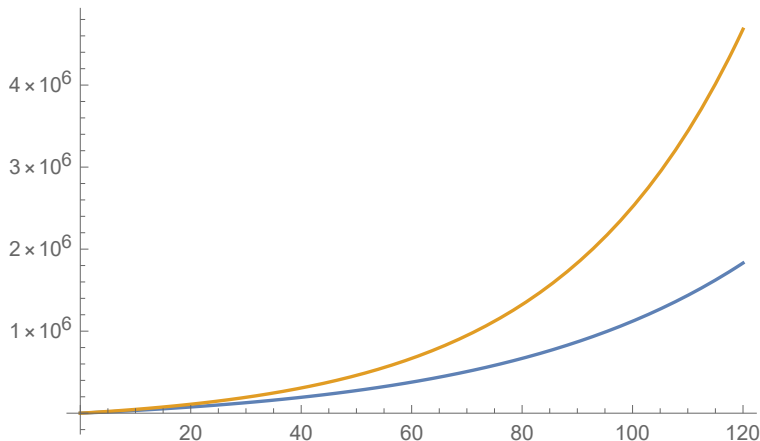
The Effect of Time



Invest \$1,000 per month at 9% for 40 years

Save up \$4,681,320.27.

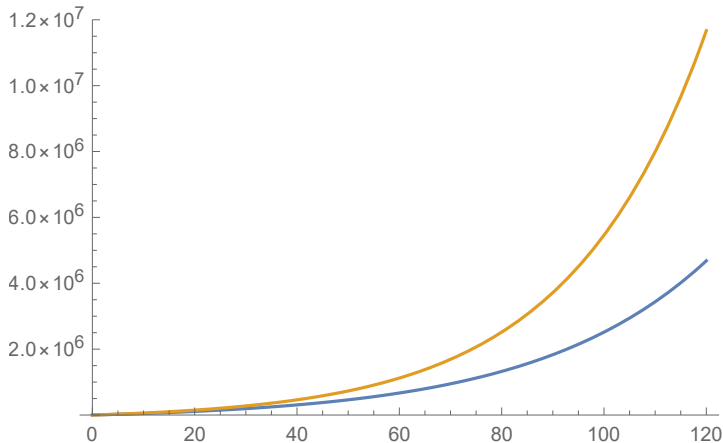
The Effect of Time



Invest \$1,000 per month at 9% for 40 years

Withdraw \$42,119.05 per month for 20 years

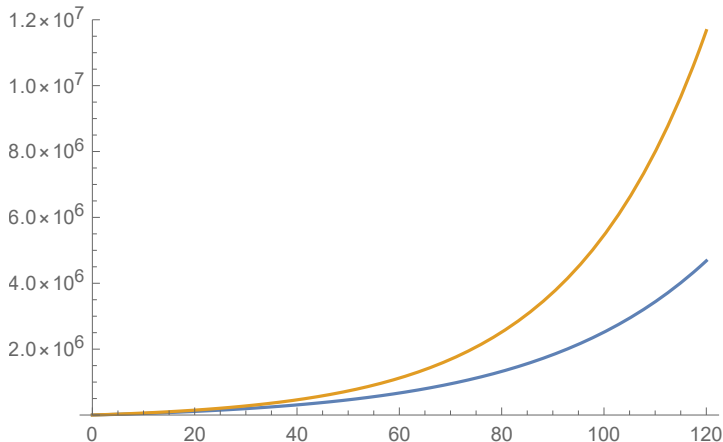
The Effect of Time



Invest \$1,000 per month at 9% for 50 years

Save up \$11,669,101.86.

The Effect of Time



Invest \$1,000 per month at 9% for 50 years

Withdraw \$104,989.94 per month for 20 years

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Example

Example (Three Deposits)

- Let the annual interest rate be 10%.
- Invest \$1000 each year for 5 years.

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The investment stage:

Year	Starting Balance	Interest	Total	Deposit	Ending Balance
1	0.00	0.00	0.00	1,000.00	1,000.00

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2	1,000.00	100.00	1,100.00	1,000.00	2,100.00

Example

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The investment stage:

Year	Starting Balance	Interest	Total	Deposit	Ending Balance
1	0.00	0.00	0.00	1,000.00	1,000.00
2	1,000.00	100.00	1,100.00	1,000.00	2,100.00
3	2,100.00	210.00	2,310.00	1,000.00	3,310.00

Example

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The investment stage:

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2	1,000.00	100.00	1,100.00	1,000.00	2,100.00
3	2,100.00	210.00	2,310.00	1,000.00	3,310.00
4	3,310.00	331.00	3,641.00	1,000.00	4,641.00

Example

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The investment stage:

Year	Starting Balance	Interest	Total	Deposit	Ending Balance
1	0.00	0.00	0.00	1,000.00	1,000.00
2	1,000.00	100.00	1,100.00	1,000.00	2,100.00
3	2,100.00	210.00	2,310.00	1,000.00	3,310.00
4	3,310.00	331.00	3,641.00	1,000.00	4,641.00
5	4,641.00	464.10	5,105.10	1,000.00	6,105.10

Annuity Formula (Building Up)

- If the payments are annual, then $k = 1$ and the formula is not too bad:

$$F = P \left(\frac{(1 + r)^t - 1}{r} \right)$$

where F is the future value of the annuity, P is the amount investment per period, r is the annual interest rate, and t is the number of years.

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- When k is greater than one, then the formula is a bit more complicated.

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- We replace r with r/k and t with kt .

Example

Example (Three Deposits)

The future value is

$$F = \frac{P((1+r)^t - 1)}{r}$$

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$$\begin{aligned} F &= \frac{P((1+r)^t - 1)}{r} \\ &= \frac{1000((1.10)^5 - 1)}{0.10} \end{aligned}$$

Example

Example (Three Deposits)

The future value is

$$\begin{aligned} F &= \frac{P((1+r)^t - 1)}{r} \\ &= \frac{1000((1.10)^5 - 1)}{0.10} \\ &= \$6,105.10. \end{aligned}$$

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Another Example

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- A person earning \$48,000 a year invests 5% of his income in a retirement account earning 7% per year for 45 years.
- How much does he have at the end of 45 years (annual payments)?

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- How much does he have at the end of 45 years (annual payments)?
- How much does he have at the end of 45 years (monthly payments)?
- What if the account earned 8% per year?

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- How much does he have at the end of 45 years (annual payments)?
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- What if the account earned 8% per year?
- What if the account earned 9% per year?

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- How much does he have at the end of 45 years (annual payments)?
- How much does he have at the end of 45 years (monthly payments)?
- What if the account earned 8% per year?
- What if the account earned 9% per year?
- What if the account earned 12% per year?

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Assignment

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- Annuity worksheet: 1 - 5.